Parametric Design Study of Electric Motor Using Multipolar Moment Matching Method Based on Model Order Reduction

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This paper presents a parametric study of the electric motor based on model order reduction (MOR). To include the effect of the parameter into the reduced model, multipolar moment matching method is used. Firstly, the high dimensional model is parameterized using the multipolar moment matching method based on Taylor series expansion and then proper orthogonal decomposition (POD) based MOR is applied to the parameterized model. The reduced model is compared with the high dimension finite element method (FEM) model of a surface mounted permanent magnet motor (SPMSM). Results prove the ability of the reduced model to provide a good approximation of the high dimensional model under the parameter variation with reduced computational cost.

***Index Terms*—Model order reduction, multipolar moment matching, parameter, finite element method, electric motor**

I. INTRODUCTION

IN THE design of electric motors, numerical methods such as the finite element method (FEM) is widely used. Whereas for a high dimensional system, FEM based analysis is time-consuming. To obtain an accurate and fast computational analysis for high dimensional systems, the proper orthogonal decomposition (POD) based model order reduction (MOR) is proposed [1]. POD based MOR uses the method of snapshots to generate the optimal basis for the reduced model. Although POD based MOR is considered as the computationally inexpensive method to analyze the electric motors, with the consideration of the parameter variations, the construction of the MORs can, however, be computationally expensive and meaningless. Because with the change of the parameters, the whole system changes and consequently previously modeled MOR is no longer valid. Thus the modeling, reduction, and analysis should be repeated every time when there is a parameter variation. To overcome this problem, parametric model order reduction (PMOR) is studied. Various PMOR techniques are documented in the literature. But these methods fail to give good approximations during multiparameter variation and effect of nonlinearities in parameters. A PMOR based on multiparameter moment matching method to analyze the coupling capacitances is presented in [2]. In this present work, POD is coupled with multiparameter moment matching method to generate a PMOR for a parametrically varying motor model. The mathematical derivation and the application on a surface mounted permanent magnet motor (SPMSM) is presented in section II. The results obtained from the reduced model are compared with the FEM analysis to validate the proposed method. The result shows the effectiveness of the proposed method in approximation the full model under parameter variation with reduced computational cost.

II. NUMERICAL METHOD AND APPLICATION

The behavior of the electric motors is studied using the magnetic vector potential formulation in FEM. The potential is approximated by the nodal basis function. Using the Galerkin method, the expression for potential can be written as

$$\boldsymbol{MU}=\boldsymbol{F} \tag{1}$$

where, $\boldsymbol{M}_{m\times m}$ is the stiffness matrix, $\boldsymbol{U}_{m\times 1}$ is the vector of the nodal potentials with m degree of freedom and $\boldsymbol{F}_{m\times 1}$ is the source vector.

A. Multiparameter moment matching method

To consider the parameter variation, (1) can be modified as

$$\boldsymbol{M}(p_1,p_2,\ldots,p_n)\boldsymbol{U}=\boldsymbol{F} \tag{2}$$

where, p_i (i=1,2,…,n) are the parameters. Using the Taylor series expansion, $\boldsymbol{M}$ in (2) can be expressed as,

$$\boldsymbol{M}(p_1,p_2,\ldots,p_n)=\tilde{\boldsymbol{M}}_0+\sum \Delta\tilde{p}_i\tilde{\boldsymbol{M}}_i+\sum_{j,k}\Delta p_j\,\Delta\tilde{p}_k\tilde{\boldsymbol{M}}_{jk}+\sum_{l,m,n}\Delta\tilde{p}_l\,\Delta\tilde{p}_m\,\Delta\tilde{p}_n\tilde{\boldsymbol{M}}_{lmn}\ldots \tag{3}$$

where, $\tilde{\boldsymbol{M}}_0$ =stiffness matrix with the initial parameters

$\Delta\tilde{p}_i$=difference between the initial and the new parameter value

$$\tilde{\boldsymbol{M}}_i=\left.\frac{\partial \boldsymbol{M}}{\partial p_i}\right|_{\bar{p}},\quad \tilde{\boldsymbol{M}}_{jk}=\left.\frac{\partial^2 \boldsymbol{M}}{\partial p_j\partial p_k}\right|_{\bar{p}}\ \ldots$$

Eq. (3) can be simplified by introducing

$$\boldsymbol{M}_i=\begin{cases}\tilde{\boldsymbol{M}}_i & i=0,1,\ldots,n\\ \tilde{\boldsymbol{M}}_{ij} & i=0,1,\ldots,n\quad j=0,1,\ldots,n\\ \tilde{\boldsymbol{M}}_{ijk} & i=0,1,\ldots,n\quad j=0,1,\ldots,n\ k=0,1,\ldots,n\\ \vdots & \vdots\end{cases}$$

$$\Delta p_i=\begin{cases}\Delta\tilde{p}_i & i=0,1,\ldots,n\\ \Delta\tilde{p}_i\,\Delta\tilde{p}_j & i=0,1,\ldots,n\quad j=0,1,\ldots,n\\ \Delta\tilde{p}_i\,\Delta\tilde{p}_j\Delta\tilde{p}_k & i=0,1,\ldots,n\quad j=0,1,\ldots,n\ k=0,1,\ldots,n\\ \vdots & \vdots\end{cases}$$

Thus (2) can be written as

$$\boldsymbol{M}(p)\boldsymbol{U}=\left(\boldsymbol{M_0}+\sum_{i=1}^{q}\Delta p_i\,\boldsymbol{M}_i\right)\boldsymbol{U}=\boldsymbol{F} \tag{4}$$

B. Proper Orthogonal Decomposition

To reduce the computational time required to solve the system (4), POD method is applied. In this method, $\boldsymbol{U}$ is approximated as $\boldsymbol{U_r}$ of size r ($r << m$) such that

$$\boldsymbol{U}=\boldsymbol{\Psi}\,\boldsymbol{U}_r \tag{5}$$

where, $\boldsymbol{\Psi}$ is the discrete projection operator, calculated by the method of snapshot. To generate the snapshot matrix, (4) is solved for n state variables and n solutions are stored in a matrix $\boldsymbol{X_s}$, called as snapshot matrix. Applying singular value decomposition (SVD), $\boldsymbol{X_s}$ can be decomposed as

$$\boldsymbol{X}_s = \boldsymbol{V}\Sigma\boldsymbol{W}^T \tag{6}$$

where, $\boldsymbol{V}_{m\times m}$ and $\boldsymbol{W}_{n\times n}$ are orthonormal matrices and $\boldsymbol{\Sigma}_{m\times n}$ is the diagonal matrix of the singular value. $\boldsymbol{\Psi}$ is constructed by the r most representative vectors of $\boldsymbol{V}$. r is selected from the singular value spectrum. Using $\boldsymbol{\Psi}$, the MOR can be deduced by combining (4) and (5)

$$\boldsymbol{M}_r(p)\boldsymbol{U}_r = \left(\boldsymbol{\Psi}^T\boldsymbol{M_0}\boldsymbol{\Psi} + \sum_{i=1}^{q}\Delta p_i(\boldsymbol{\Psi}^T\boldsymbol{M}_i\boldsymbol{\Psi})\right)\boldsymbol{U}_r = \boldsymbol{F}_r \tag{7}$$

where, $\boldsymbol{M}_r = \boldsymbol{\Psi}^T\boldsymbol{M}\boldsymbol{\Psi}$, $\boldsymbol{F}_r = \boldsymbol{\Psi}^T\boldsymbol{F}$. The overall computational operation can be divided into (i) Initial parameter model based on FEM, (ii) parameterization using Taylor series (iii) POD based MOR.

C. Application and Result

In terms of application, a 12 pole 9 slot SPMSM, shown in Fig. 1(a) is studied. The system is analyzed without connecting to electric source. The airgap magnetic flux density is considered for analysis. One third of the motor is modeled for symmetry in structure. The analysis can be divided into (i) single parameter variation and (ii) multiparameters variation. For the analysis, two parameters are considered, namely, residual magnetic flux density, B_r of the PM and magnet thickness t_{mag}. The initial values of B_r and t_{mag} are taken as 0.8 T and 10 mm, respectively. In single parameter case, only B_r is varied. Firstly, with the initial value of B_r, FEM analysis of the SPMSM is carried out. Next, B_r value is changed to 1.2 T and FEM is done. Under these two cases, airgap gap flux density variation is computed for one pole pitch movement of the motor. Using these data, the Taylor series expansion of (4) is generated for parameterization. The coefficients can be derived as

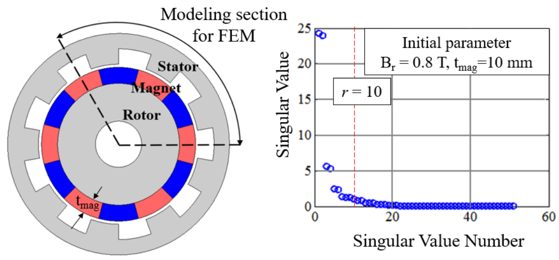
$$\widetilde{\boldsymbol{M}}_0 = \boldsymbol{M}_{\mathrm{FEM}}(\boldsymbol{B_r} = 0.8\mathrm{T}) \tag{8}$$

$$\widetilde{\boldsymbol{M}}_1 = \frac{1}{0.4}\left(\boldsymbol{M}_{\mathrm{FEM}}(\boldsymbol{B_r} = 1.2\mathrm{T}) - \widetilde{\boldsymbol{M}}_0\right),\ \Delta \boldsymbol{p} = 0.4 \tag{9}$$

Then to obtain reduced model, (6) is applied on (8) and (9). From the SVD of (8) and (9), the singular value spectrum can be plotted as shown in Fig. 1(b) and 2(a). From the plot, the reduced model is created only by using r equal to 10. For multiparameter variation, both B_r and t_{mag} are considered. FEM is carried out with varying the parameters to 1.2 T and 11 mm. The parameterization process is same as mentioned above. The basis function can be constructed by selecting r as 5, as shown in Fig. 2(b). The process of parameterization and MOR are carried out in MATLAB platform. Once the reduced model is generated, the computation of the airgap flux density variation with respect to a new value of B_r and t_{mag} can easily be calculated by the reduced model in MATLAB platform, without doing repeated high dimensional full model FEM analysis.

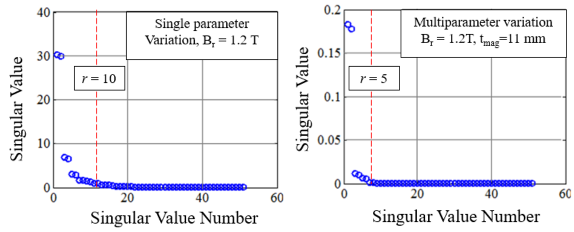
With the PMOR model, different cases of single and multiparameter variations are studied. For single parameter case, B_r values of 1 T and 1.5 T are chosen. The results obtained by reduced model is compared with that of the FEM models. Fig. 3(a) shows the comparison between the FEM and proposed PMOR data for the airgap flux density. The average percentage of approximation difference between the data obtained by FEM and PMOR is less than 0.5%. In multiparameter case, B_r and t_{mag} are taken as 1 T and 10.5 mm. The comparison between the FEM and PMOR models are presented in Fig. 3(b). The average approximation difference less than 1%

In terms of computational cost, the proposed method is cost effective, because once the parametric effect is added to the reduced model, there is no need to analyze the whole high order system repeatedly, every time with the change of the parameter, like in the case of FEM. One needs to solve only a small matrix of size r, where $r << m$. Also, the PMOR shows good approximation result with multiparameter variation. In the future extended manuscript, the further detail analysis of the effect of nonlinearity on the approximated data will be included.



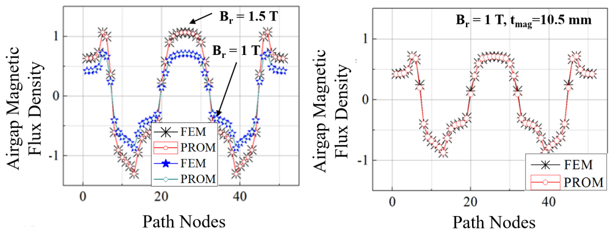
(a) (b)

Fig. 1. (a) 12 pole 9 slot SPMSM. (b) singular value spectrum for initial model



(a) (b)

Fig. 2. Singular value spectrum for (a) single parameter (b) multiparameter variation



(a) (b)

Fig. 3 airgap flux density distribution for (a) single parameter (b) multiparameter variation

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